Corrections and additions to "An Introduction to State Space Time Series Analysis"
by Jacques J.F. Commandeur and Siem Jan Koopman

Six corrections are listed below, p. and l. indicate page and line numbers (negative numbers should be counted from the bottom), → indicates "should read".

1. p. 33, l. –4: Since it follows from (4.1) that $\gamma_1 = \gamma_{1,1}$, $\gamma_2 = \gamma_{1,2} = \gamma_{2,1}$, and $\gamma_3 = \gamma_{1,3} = \gamma_{2,2} = \gamma_{3,1}$, we also treat γ_1 , γ_2, and γ_3 as fixed and unknown coefficients. → Since it follows from (4.1) that $\gamma_1 = \gamma_{1,1}$, $\gamma_4 = \gamma_{2,1}$, and $\gamma_3 = \gamma_{3,1}$, we also treat the initial values of γ_1, γ_4, and γ_3 as fixed and unknown coefficients.

2. pp. 80-81: The last equation on page 80 and the first equation on page 81 should read:

$$y_t = \mu_1 + (t-1)\nu_1 + \beta_1 \sum_{i=1}^{t-2} (t-i-1)x_i + \varepsilon_t$$

   with

$$\sum_{i=1}^{t-2} (t-i-1)x_i = 0 \quad \text{when} \quad t = 1, 2.$$

3. p. 85, l. –6, eq. (8.5): $a_{t+1} = a_t + K_t(y_t - z_t' a_t)$ → $a_{t+1} = T_t a_t + K_t(y_t - z_t' a_t)$

4. p. 112, l. 6: For multivariate models with $p > 2$ → For multivariate models with $p \geq 2$

5. p. 113, l. 6: *b* and *c* → *a* and *b*

6. p. 144: the formula for the log-likelihood function on this page should read

$$\text{dLogLik} = \log L(y \mid \psi) = -\frac{np}{2}\log(2\pi) - \frac{1}{2}\sum_{t=d+1}^{n}\left(\log|F_t| + v_t' F_t^{-1} v_t\right)$$

Whenever required, we will elaborate upon the corrections mentioned above. So far, one correction (i.e., 1.) needs further explanation:

ad 1. The local level model with deterministic dummy seasonal for quarterly data can be written as:

$$
\begin{aligned}
&y_t = \mu_t + \gamma_t + \varepsilon_t \\
&\mu_{t+1} = \mu_t + \xi_t \\
&\gamma_{1,t+1} = -\gamma_{1,t} - \gamma_{2,t} - \gamma_{3,t} \\
&\gamma_{2,t+1} = \gamma_{1,t} \\
&\gamma_{3,t+1} = \gamma_{2,t}
\end{aligned} \tag{4.1}
$$

(see page 32).
Writing out the terms for the seasonal component in these equations time point by time point, it can be verified that

γ_t	t = 1	2	3	4	5	6	etc.
$\gamma_{1,t}$	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	
$\gamma_{2,t}$	Q_4	Q_1	Q_2	Q_3	Q_4	Q_1	
$\gamma_{3,t}$	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4	

where Q_j denotes the value of the seasonal for quarter j = 1, …, 4, while for $\gamma_{i,t}$

$\gamma_{i,t}$	t = 1	2	3	4	5	etc.
i = 1	$\gamma_{1,1}$	$\gamma_{1,2}$	$\gamma_{1,3}$	$\gamma_{1,4}$	$\gamma_{1,5}$	
2	$\gamma_{2,1}$	$\gamma_{2,2} = \gamma_{1,1}$	$\gamma_{2,3} = \gamma_{1,2}$	$\gamma_{2,4} = \gamma_{1,3}$	$\gamma_{2,5} = \gamma_{1,4}$	
3	$\gamma_{3,1}$	$\gamma_{3,2} = \gamma_{2,1}$	$\gamma_{3,3} = \gamma_{2,2} = \gamma_{1,1}$	$\gamma_{3,4} = \gamma_{2,3} = \gamma_{1,2}$	$\gamma_{3,5} = \gamma_{2,4} = \gamma_{1,3}$	

It follows that $\gamma_1 = \gamma_{1,1}$, $\gamma_4 = \gamma_{2,1}$, and $\gamma_3 = \gamma_{3,1}$, and that the initialisation procedure for a quarterly seasonal requires initial estimates for quarters 1, 4, and 3, respectively.

We are grateful to Jørn Toft Bysveen, Noor Wahida, and Nathaniel Derby for pointing out corrections 1, 2, and 3 to us, and to Steinar Veka for pointing out correction 6.

We also would like to thank Tomonori Matsuki and Dongling Huang for their interest in our book. Here is an answer to your questions concerning the way in which the value of the log-likelihood function is evaluated in SsfPack, the software that we used for fitting the state space models discussed in the book:

In our book we report the maximised log-likelihood values for most estimated models. The log-likelihood functions that we compute belong to the class of marginal log-likelihood functions. In models with non-stationary variables and fixed regressions in the state vector, we require to specify diffuse initial conditions since the initial distribution of such variables is not properly defined. In effect we compute the likelihood function of a linearly transformed data-set that does not rely on the initial conditions.

More generally, the marginal likelihood is defined as the likelihood function of a transformation of the data vector. The transformation is not unique. The diffuse likelihood is a marginal likelihood for a specific data transformation. For different models, other transformations are implied and therefore we cannot strictly use it for

model comparison although the presented log-likelihood comparisons in our book are still indicative. Furthermore, the diffuse log-likelihood function loses its interpretation as a joint probability in logs. The motivation of our use of diffuse or marginal likelihood functions is that the resulting parameter estimates have better small sample properties compared to other likelihood functions. In a paper by Francke, de Vos and Koopman (2008), a more detailed and technical discussion can be found on likelihood functions for time series models in state space form with many references to the literature on this subject. This is the link to the just mentioned paper:
http://www.tinbergen.nl/discussionpapers/08040.pdf

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